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Continuous-variable quantum teleportation of even and odd coherent states through varied gain channels*

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This paper has investigated quantum teleportation of even and odd coherent states in terms of the EPR entanglement states for continuous variables. It discusses the relationship between the fidelity and the entanglement of EPR states, which is characterized by the degree of squeezing and the gain of classical channels. It shows that the quality of teleporting quantum states also depends on the characteristics of the states themselves. The properties of teleporting even and odd coherent states at different intensities are investigated. The difference of teleporting two such kinds of quantum states are analysed based on the quantum distance function.

Keywords: quantum teleportation, odd and even coherent state, fidelity
PACC: 4205, 7335, 7215R

1. Introduction

Since the presentation of quantum teleportation in 1993,[1] it has been extensively investigated both in theory and experiment and has played an important role in the quantum information science (QIS). Continuous-variable (CV) teleportation, which was first proposed by Vaidman[2] is a significant issue in QIS.[3] CV quantum teleportation was first demonstrated by Furusawa *et al* experimentally in 1998[4] and later improved.[5,6] Recently, it was reported that the CV teleportation of a squeezed states was demonstrated.[7] Based on multi-entanglements the network of quantum information transportation appears to be a reality.[8] From its first accomplishment for coherent states to present squeezed states, quantum teleportation for other quantum states can be realized and the quality of quantum teleportation for various typical quantum states is definitely important since we have to face lots of quantum states with different nonclassical properties for quantum information processes.

In this paper we have investigated the teleportation of even and odd coherent states (EOCS), which are the non-classical states and have many interesting properties such as squeezing, anti-bunching effect, etc.[9] The general properties and generation of EOCS were studied by many authors.[10−12] As the compositions of coherent states of light fields, the EOCS are important in quantum information processing and fundamentals of quantum optics, especially for the study of Schrödinger states[13,14] and in the study of entanglement, quantum measurement, nonlocality and quantum information.[15−17]

Although the quantum teleportation with lossy channels has been discussed,[18] the gain of the classical channels is another important factor which can be easily adjusted and may affect the results of the teleportation. By using entangled two-mode squeezed states,[19] i.e. the EPR states for CV, we obtain the fidelity of teleporting EOCS and discuss the dependence of the fidelity on the entanglement parameter of the EPR pairs as well as the gain of the classical channels. It shows that the higher entanglement is necessary for teleporting a quantum state with nonclassicality and proper classical gain is also needed for best quality of teleportation.

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2. Basic model

The basic configuration of the quantum teleportation is shown in Fig.1. The EPR pairs, stated as $\rho_{\rm EPR}$, are separated by a beam splitter. One part, EPR1 is going to overlap with an EOCS, stated as input state $\rho_{\rm in}$, on the 50/50 beam splitter. The quadrature phases, X and Y, of the superposition states are measured by two homodyne detection systems. The results are sent to Bob's station, which consists of two modulators corresponding to the two quadratures, X and Y, respectively. The transmitted light superposes with the second part of the EPR pairs, EPR2, and then the already dead input states are recovered and appeared as an output states.

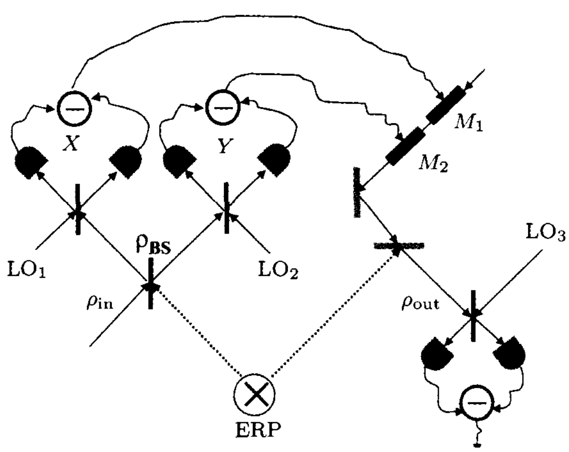

Fig.1. Schematic configuration of quantum teleportation.

The EPR pairs of CV is the two mode squeezed vacuum states, which can be expressed by Fock states[20-22]

$$\rho_{\rm EPR}=(1-\lambda^2)\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}(-\lambda)^{n+n'}|n,n\rangle_{1,2}\langle n',n'|, \quad (1)$$

where λ is the entanglement parameter of EPR pair which is directly related to the degree of squeezing parameter τ

$$\lambda=\tanh(\tau)\qquad 0<\lambda<1\,. \quad (2)$$

Assume that the input state is the EOCS $|\psi_\pm\rangle$, its density operator $\rho_{\rm in}$ is

$$\rho_{\rm in}=|\psi_\pm\rangle\langle\psi_\pm|,$$

$$|\psi_\pm\rangle=N_\pm(|\beta\rangle\pm|-\beta\rangle),$$

$$|\beta\rangle={\rm e}^{-\frac{1}{2}|\beta|^2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|n\rangle,$$

$$N_\pm=1/\sqrt{2[1\pm\exp(-2|\beta|^2)]}\,,$$

$$(n=0,1,2,\ldots)\,. \quad (3)$$

The initial state of whole system is then

$$\rho_0=\rho_{\rm EPR}\otimes\rho_{\rm in}, \quad (4)$$

where $N_\pm$ in Eq.(3) is an unitary factor. N_+ and N- correspond to the even coherent state $|\beta\rangle+|-\beta\rangle$ and the odd coherent state $|\beta\rangle-|-\beta\rangle$ respectively. $|\beta|^2$ is the average photon number of coherent states $|\beta\rangle$ and $|-\beta\rangle$.

The state of the whole system after the 50/50 beamsplitter can be described as

$$\rho_{\rm BS}=U_{\rm BS}^{+}\rho_0U_{\rm BS}, \quad (5)$$

where $U_{\rm BS}$ is the unitary operator of the 50/50 beamsplitter which is

$$U_{\rm BS}={\rm e}^{\frac{\pi}{4}(\hat a_{\rm in}\hat a_1^{+}-\hat a_{\rm in}^{+}\hat a_1)}. \quad (6)$$

When the measurements of amplitude quadrature X and phase quadrature Y are done, the measured fields collapse to their eigenstates and, according to the measurement theory, the rest of the system is collapsed to a new state, $\rho_{\rm M}$[23]:

$$\rho_{\rm M}=\frac{{\rm Tr}_{\rm in,1}\{\rho_{\rm BS}|X\rangle_{\rm in}\langle X|\otimes|Y\rangle_1\langle Y|\}}{{\rm Tr}_2{\rm Tr}_{\rm in,1}\{\rho_{\rm BS}|X\rangle_{\rm in}\langle X|\otimes|Y\rangle_1\langle Y|\}}, \quad (7)$$

where

$$X=\frac{1}{2\sqrt{2}}(a_{\rm in}+a_1+a_{\rm in}^{+}+a_1^{+}),$$

$$Y=\frac{1}{2\sqrt{2}\,{\rm i}}(a_{\rm in}+a_1-a_{\rm in}^{+}-a_1^{+}). \quad (8)$$

The measured classical signals are transmitted from Alice to Bob by classical channels and modulate the beam at Bob's station, then on the second beamsplitter with very high reflectivity, together with the second part of the EPR pairs, EPR2, accomplishes the process of a displacement transformation $D[\sqrt{2}g(X-{\rm i}Y)]$, where g is the classical gain for the classical channels.

From Eqs.(1) to (8) we get the output density operator of the system $\rho_{\rm out}$:

$$\rho_{\rm out}=\sqrt{\frac{2(1-\lambda^2)}{\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}{\rm d}X{\rm d}Y{\rm e}^{-(X^2+Y^2)}$$

$$\times D(\sqrt{2}g(X-{\rm i}Y)|\Phi\rangle\otimes{\rm h.c.}),$$

$$|\Phi\rangle = f(\gamma,\psi)\hat{D}(\lambda[\gamma - \sqrt{2(X-\mathrm{i}Y)}])|0\rangle,$$

$$f(\gamma,\psi) = \frac{1}{\pi}\int_{-\infty}^{\infty} \mathrm{d}^2\gamma \mathrm{e}^{-\frac{1}{2}|\gamma|^2} \times \mathrm{e}^{\sqrt{2}\gamma(X+\mathrm{i}Y)}\mathrm{e}^{\frac{1}{2}\lambda^2|\gamma-\sqrt{2}(X-\mathrm{i}Y)|^2}\langle\gamma|\psi\rangle_{\mathrm{in}}, \quad (9)$$

where $|\gamma\rangle$ is a coherent state which plays an important role in the transformation process and h.c. denotes the conjugation. We have used the relation

$$\sum_{\mathrm{in},1,2} \rho_{\mathrm{BS}}|X\rangle\langle X| \otimes |Y\rangle\langle Y| = 1. \quad (10)$$

The quality of characterizing the quantum teleportation process is given by the fidelity which is defined as

$$F = \mathrm{Tr}(\rho_{\mathrm{in}} \cdot \rho_{\mathrm{out}}). \quad (11)$$

Substituting Eqs.(3) and (9) into Eq.(11), we get the fidelities $F_{\pm}$ of EOCS, respectively:

$$F_{\pm} = \frac{2(1-\lambda^2)}{kN_-}\Big\{\mathrm{e}^{\frac{-(1-g)^2(1-\lambda^2)|\beta|^2}{k}} + \mathrm{e}^{\frac{-(1+g)^2(1-\lambda^2)|\beta|^2}{k}} - \mathrm{e}^{-2(1-\lambda)|\beta|^2}\mathrm{e}^{\frac{-(1-g)^2(1+\lambda)^2|\beta|^2}{k}} \pm 2\mathrm{e}^{-2|\beta|^2}\Big[\mathrm{e}^{\frac{(1-g^2)(1-\lambda^2)|\beta|^2}{k}} + \mathrm{e}^{\frac{-(1-g^2)(1-\lambda^2)|\beta|^2}{k}}\Big] + \mathrm{e}^{-2(1+\lambda)|\beta|^2}\mathrm{e}^{\frac{-(1+g)^2(1-\lambda)^2|\beta|^2}{k}}\Big\}, \quad (12)$$

where

$$k = 1 + g^2 - 2g\lambda. \quad (13)$$

This clearly shows that the fidelities of quantum teleportation depend on not only the classical gain g of the classical channels, and the parameter λ which features the entanglement of EPR source, but also the average photon number $|\beta|^2$ of coherent states and the teleported states themselves. For perfect EPR source, $\lambda = 1$. If the gain is 1, one can realize ideal quantum teleportation, i.e. $F_{\pm} = 1$. In real situations, λ is less than 1, and there exists an optimal gain for certain intensity of $|\beta|^2$ for optimal fidelities.

3. Numerical results and discussions

The most important factor that affects the quality of quantum teleportation is the entanglement, which is characterized by the squeezing parameter λ. Fig.2 shows the fidelity F_- for even coherent states as functions of the gain g and the parameter λ, where we have chosen $|\beta|^2 = 25$. It is very reasonable that the higher the squeezing parameter, the better the fidelity. With certain degree of entanglement, there exists an optimal gain which is, in general, not unity any more, but less than 1. Only in the case of $\lambda = 1$ and $g = 1$, can one have perfect quantum teleportation.

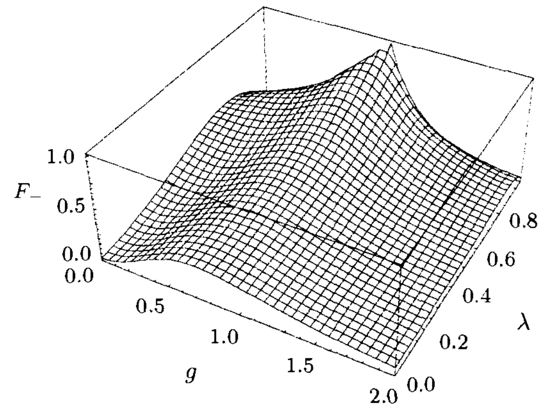

Fig.2. F_- as functions of g and λ ($|\beta|^2 = 25$).

Characterization of the entanglement of a quantum system is still an interesting issue under investigation. For the CV EPR source which is considered here, one can also use the von Neumann entropy[24-26] to describe the degree of entanglement, which is[27]

$$S_{\lambda} = \mathrm{Tr}[\rho_{\mathrm{vacum}}\log_2(\rho_{\mathrm{vacum}})] = \frac{1}{1-\lambda^2}\log_2\Big(\frac{1}{1-\lambda^2}\Big) - \frac{\lambda^2}{1-\lambda^2}\log_2\Big(\frac{\lambda^2}{1-\lambda^2}\Big) \quad (\lambda = \mathrm{th}(r)). \quad (14)$$

Figure 3 shows the fidelities F_+ and F_- as a function of entanglement degree S_{λ}. The fidelities increase as the von Neumann entropy increases. Higher intensity of the input light corresponds to better fidelities, because stronger and stronger beam will eventually reach the classical limit of light fields. There is no much difference between F_+ and F_- in the case of relative high intensity, where we have chosen $g = 1$

and $|\beta|^2 = 25$. But when the average photon number is small, $F_-$is smaller than F_+on the same conditions which indicates that on the low light level, the quantum features of EOCSs appear different nonclassical properties. Fig.4 shows the results in the case of $g = 1$ and $|\beta|^2 = 1/4$. When the entanglement becomes perfect the fidelities always approach to 1 and are independent on the input states.

Why the results of teleportation for EOCS are different and dependent on the intensities? We have to investigate the difference of these two states. We can use the well known distance function d,[28] the larger the distance, the more different the two states. Let us check the distances between the even (odd) coherent states and the coherent states.

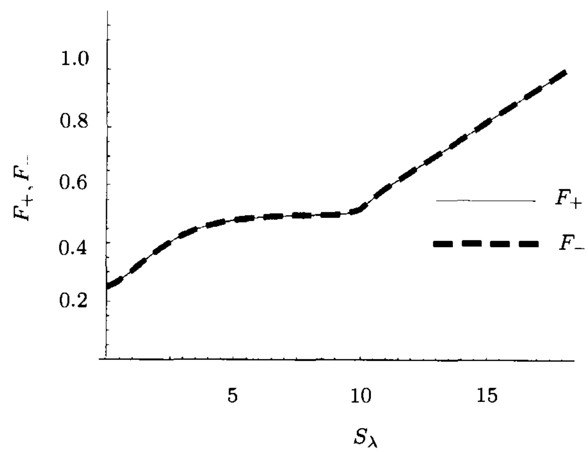

Fig.3. Change of F_+ and F_- with $S_\lambda(|\beta|^2 = 25, g = 1)$.

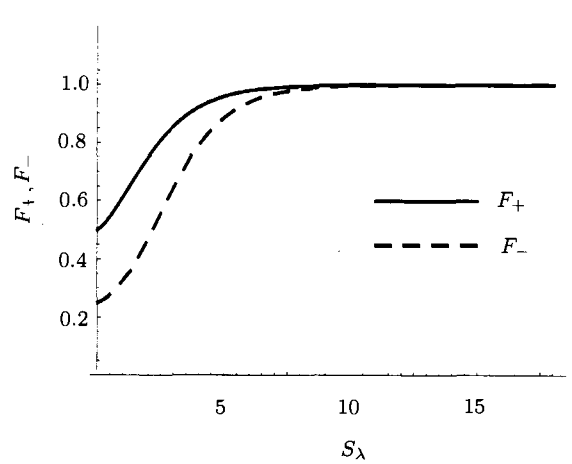

Fig.4. Change of fidelities with S_λ ($|\beta|^2 = 1/4$, $g = 1$).

Suppose the density operator of coherent states, which makes up the even (odd) coherent states, is $\rho = |\beta\rangle\langle\beta|$. The density operators of the EOCS are ρ_e and ρ_o respectively. We have

$$\rho_e = N_+^2(|\beta\rangle + |-\beta\rangle)(\langle\beta| + \langle-\beta|), \tag{15}$$

and

$$\rho_o = N_-^2(|\beta\rangle - |-\beta\rangle)(\langle\beta| - \langle-\beta|). \tag{16}$$

It is easy to prove that

$$\mathrm{Tr}(\rho_o\rho) = \frac{1 - 2\exp(-2|\beta|^2) + \exp(-4|\beta|^2)}{2[1 - \exp(-2|\beta|^2)]},$$

$$\mathrm{Tr}(\rho_e\rho) = \frac{1 + 2\exp(-2)|\beta|^2 + \exp(-4|\beta|^2)}{2[1 + \exp(-2|\beta|^2)]}. \tag{17}$$

The distance between two pure quantum states ρ_1 and ρ_2 is [28]

$$d = \sqrt{1 - \mathrm{Tr}(\rho_1 \cdot \rho_2)}\,. \tag{18}$$

Let $\rho_1 = \rho$, $\rho_2 = \rho_e$ or ρ_o, we get the distance d_e, d_o between even coherent state ρ_e, odd coherent state ρ_o and the general coherent state ρ respectively,

$$d_e = \left\{\frac{1}{2}[1 - \exp(-2|\beta|^2)]\right\}^{1/2},$$

$$d_o = \left\{\frac{1}{2}[1 + \exp(-2|\beta|^2)]\right\}^{1/2}. \tag{19}$$

Figure 5 shows the distances of d_e, d_o as functions of the average photon number $|\beta|^2$ at relative low intensity level. When the mean photon number is less than one, even coherent state is much more closer to coherent state than that of the odd coherent state, while for large number of photons, both even and odd coherent states have the same distance to coherent state, $1/\sqrt{2}$. We can explain the previous results of the fidelities at large number of photons. Figure 6 shows the fidelities F_+ and F_- as a function of the mean photon number $|\beta|^2$ ($g = 1$ and $\lambda = 0.82$). Clearly, the fidelity of teleporting even coherent state is higher than that of the odd coherent state. When the intensities are getting higher, the fidelities go to the same.

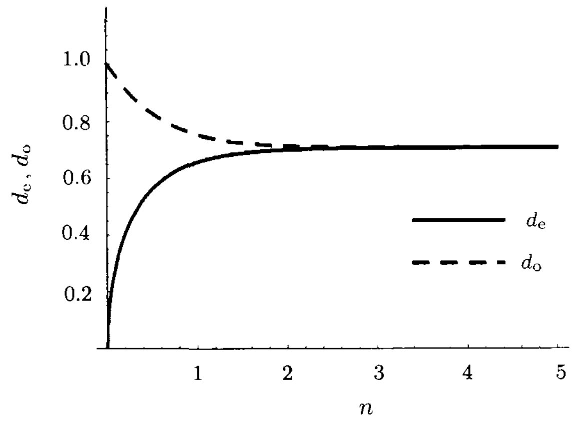

Fig.5. d_e and d_o as function of mean photon number.

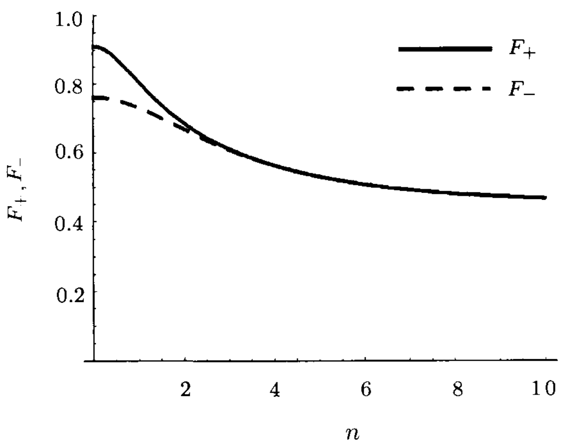

Fig.6. F_+ and F_- versus average photon number $|\beta|^2$, where $g = 1$ and $\lambda = 0.82$.

Actually, when the entanglement of the EPR source is $0(\lambda = 0)$, and classical gain is 1, the fidelity corresponds to the boundary of classical and quantum teleportation.[15] Clearly, if $\lambda = 0$, $g = 1$ and $|\beta|^2 = 0$, the fidelities are 1/4 and 1/2 for odd and even coherent states respectively, which are the classical limits. The EOCS can actually be expressed in Fock state basis,[8]

$$|\psi_+\rangle = \Big(\frac{1}{\cosh|\beta^2|}\Big)^{1/2} \sum_{m=0}^{\infty} \frac{\beta^{2m}}{\sqrt{(2m)!}}|2m\rangle,$$

$$|\psi_-\rangle = \Big(\frac{1}{\sinh|\beta^2|}\Big)^{1/2} \sum_{m=0}^{\infty} \frac{\beta^{2m}}{\sqrt{(2m+1)!}}|2m+1\rangle, \quad (20)$$

where m is the photon number. It is clear that when average photon number close to 0, the even coherent state is approaching to a vacuum state, which can be thought as the classical-quantum boundary, whereas the odd coherent state is approaching single photon state, which is a typical nonclassical state.[16] Reference [22] has discussed the criterions of teleporting the Fock states with photon number m. It gave

$$F_{\text{Fock}} = \frac{(2m)!}{2^{2m+1}(m!)^2}. \quad (21)$$

This result gives 1/2 and 1/4 exactly for vacuum state ($m = 0$) and single photon state ($m = 1$) respectively.

Figures 7 and 8 show the fidelities of odd and even coherent states versus the gain and entanglement for average photon number $|\beta|^2 = 1$. The big difference of the fidelities for poor entanglement and the similarity for strong entanglement are clearly shown from these 3D figures.

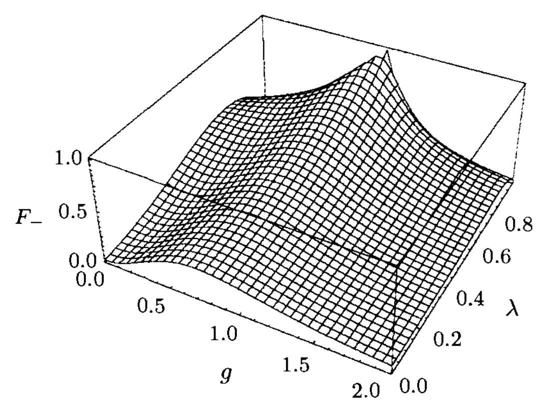

Fig.7. F_- versus g and λ ($|\beta|^2 = 1$).

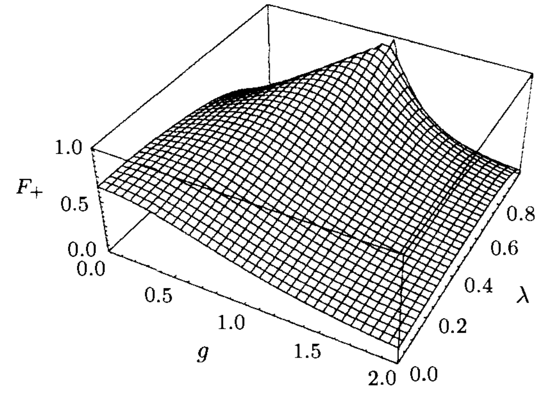

Fig.8. F_+ versus g and λ ($|\beta|^2 = 1|$).

4. Conclusion

We have discussed the CV quantum teleportation of EOCS in the Schrödinger picture. The quality of EPR source, the gain of the classical channel and the intensity of the states are extensively investigated. This shows that the fidelities of EOCS at low photon numbers are very different from those at large number of photons. The intrinsic reason is discussed by the distance of quantum states and the result can be used to explain for different mean photon numbers. The odd coherent state is more difficult to be teleported than that of the even coherent state in any case. This discussion is helpful to understand the process of quantum teleportation for various quantum states and look for the best conditions of realizing optimal teleportation in the real situations.

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